SOUND ABSORPTION NEAR THE BOUNDARY DIVIDING TWO LIQUIDS

V. A. Murga

UDC 534.2:532

It is well known that sound absorption in finite media is caused mainly by fluid viscosity and thermal conductivity. Kirchhoff [1] developed a general theory describing the mechanism of such absorption and applied it to the particular case of sound propagating in tubes. Rayleigh [2] used Kirchhoff's theory to study sound absorption by a porous wall with normal incidence of the sound wave. Konstantinov [3] also used Kirchhoff's theory to solve the problem of sound absorption by a rigid, isothermal (with infinite thermal conductivity) and a thermally insulating plane wall with arbitrary angle of sound-wave incidence. A natural extension of these efforts is a study of sound absorption on the boundary dividing two liquids. Aside from its scientific interest, such a problem is of practical significance, for example, in hydroacoustics or in creating methods for visualization of sound in gases and liquids [4]. The present study will attempt to solve this problem. The results can be applied to both liquid and solid (resinlike) materials.

1. In the absence of sonic oscillations let the boundary dividing the two liquids form a horizontal plane, so that for brevity we may refer to the upper and lower media. In the upper medium a planar sinusoidal sound wave falls on the phase boundary. We introduce a Cartesian coordinate system such that the phase boundary lies in the plane xz, the incident, reflected, and refracted rays lie in the plane xy, and the y axis is directed into the upper medium. The temperature and velocity fields in each medium are described by linearized hydromechanics equations

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho}\operatorname{grad} P + \nu\Delta\mathbf{v} + \frac{1}{3}\nu\operatorname{grad}(\operatorname{div}\mathbf{v}),\ \frac{\partial s}{\partial t} + \operatorname{div}\mathbf{v} = 0,$$
$$\frac{\partial T}{\partial t} = \frac{\gamma-1}{\alpha}\frac{\partial s}{\partial t} + \frac{\varkappa}{\rho c_V}\Delta T,\ s = \frac{\gamma}{c^2}\frac{P}{\rho} - \alpha T, \tag{1.1}$$

where $\mathbf{v}$ is velocity; t is time; ρ, density; ν, kinematic viscosity; $\varkappa$, thermal conductivity; $\gamma = c_P/c_V$; c_P and c_V are the specific heats of the liquid at constant pressure and volume, respectively; α is the thermal expansion coefficient; c is the speed of sound (Laplacian); s is the acoustic compression of the medium; T and P are the acoustic temperature and pressure.

In the future we will assume that in both media the conditions

$$(\nu\omega/c^2)^{1/2} \ll 1,\quad (\chi\omega/c^2)^{1/2} \ll 1 \tag{1.2}$$

are satisfied, where χ is the thermal diffusivity.

The solution of Eq. (1.1) corresponding to the proposed problem must have a form such that the dependence on time is given by a factor $\exp(-ht)$, and the dependence on longitudinal coordinate x by a factor $\exp(mx)$, where $h = i\omega$, $m = ik\sin\theta$, ω is the angular frequency of the oscillations, $k = \omega/c$, θ is the angle of incidence (reflection); we will omit these factors below. Such a solution for temperature, pressure, and velocity components can be written for the upper medium to an accuracy of terms insignificant for the present problem in the form:

$$P = Q_2\rho c^2,\quad u = AQ + c\sin\theta\cdot Q_2,$$
$$v = A\frac{m\nu}{h}\frac{dQ}{dy} + A_1(\gamma-1)\chi\frac{dQ_1}{dy} + \frac{c\sin\theta}{m}\frac{dQ_2}{dy},\quad T = \frac{\gamma-1}{\alpha}(A_1Q_1 + Q_2). \tag{1.3}$$

Here u and v are the components of the velocity $\mathbf{v}$ along the x and y axes; $Q = \exp\left(i\sqrt{\frac{h}{\nu}}\,y\right)$; $Q_1 = \exp\left(i\sqrt{\frac{h}{\chi}}\,y\right)$; $Q_2 = k^2[\exp(-ik\cos\theta\cdot y) + A_2\exp(ik\cos\theta\cdot y)]$; A, A_1, A_2 are undefined constants.

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 6-9, January-February, 1984. Original article submitted November 2, 1982.

 0021-8944/84/2501-0004$08.50 © 1984 Plenum Publishing Corporation

The function Q_2 has the sense of a set of incident and reflected waves at a distance from the phase boundary sufficiently great that the effects of temperature and dynamic boundary layers may be neglected; A_2 is the reflection coefficient.

For the lower medium the expressions for the temperature and velocity are analogous to Eq. (1.3), with

$$Q' = \exp\left(-i\sqrt{\frac{h}{\nu'}}\,y\right), \quad Q_1' = \exp\left(-i\sqrt{\frac{h}{\chi'}}\,y\right),$$
$$Q_2' = A_2' k'^2 \exp(-ik'y\cos\theta').$$

Here the primes indicate values for the lower medium; θ' is the angle of refraction; Q_2' has the sense of the wave which passes through the medium; A_2' is the transmission coefficient. Moreover, we assume here the absence of complete internal reflection of sound from the phase boundary (subcritical angle of incidence). The case of supercritical angle of incidence will be considered below.

In the linear approximation boundary conditions on the perturbed phase boundary can be replaced by conditions on an unperturbed plane. Thus, at $y = 0$ the following expressions must be satisfied [with consideration of Eq. (1.2)]:

$$u = u', \quad v = v', \quad T = T', \quad \varkappa\partial T/\partial y = \varkappa'\partial T'/\partial y, \quad \mu\partial u/\partial y = \mu'\partial u'/\partial y, \qquad (1.4)$$
$$P = P'.$$

It is assumed here that the static temperature and pressure in both media are the same and that $\mu = \nu\rho$.

Substituting Eq. (1.3) in boundary conditions (1.4), we obtain a system of algebraic equations in the unknown constants A, A_1, A_2, A', A_1', A_2'. Solution of this system gives an expression for the reflection coefficient

$$A_2 = \frac{\cos\theta - (\rho c/\rho' c')\cos\theta' - M}{\cos\theta + (\rho c/\rho' c')\cos\theta' + M} \qquad (1.5)$$

and transmission coefficient

$$A_2' = \frac{(2\rho/\rho')\cos\theta}{\cos\theta + (\rho c/\rho' c')\cos\theta' + M}, \qquad (1.6)$$

where

$$M = \frac{1-i}{\sqrt{2}}\left[\sin^2\theta\cdot\left(\frac{\nu\omega}{c^2}\right)^{1/2}\frac{(1-\rho/\rho')^2}{1+\sqrt{\mu\rho/\mu'\rho'}} + (\gamma-1)\left(\frac{\chi\omega}{c^2}\right)^{1/2}\frac{(1-\rho c_p\alpha'/\rho' c_p'\alpha)^2}{1+\sqrt{\varkappa\rho c_p/\varkappa'\rho' c_p'}}\right]. \qquad (1.7)$$

If we neglect viscosity and thermal conductivity (take $M = 0$), then Eqs. (1.5), (1.6) transform to Rayleigh's expression [2].

If $\rho/\rho' \to 0$, then Eq. (1.5) transforms to Konstantinov's expression [3] for the case of a rigid wall with infinite thermal conductivity. If $\rho/\rho' \to 0$, but $\varkappa\rho/\varkappa'\rho' \to \infty$, then Eq. (1.5) also transforms to Konstantinov's result [3], but for the case of a rigid thermally insulated wall.

The absorption coefficient, defined as the ratio of the mechanical energy dissipated per unit time in the boundary layer near the phase boundary to the energy flux incident from the upper medium, is equal to

$$D = 1 - A_2\bar{A}_2 - \frac{\rho' c\cos\theta'}{\rho c'\cos\theta}A_2'\bar{A}_2'$$

(we deal here with the time-averaged absorption coefficient, and the bar denotes a complex conjugate value). Using Eqs. (1.5), (1.6), we obtain

$$D = 4X/[(X+Y+1)^2+1], \qquad (1.8)$$

where

$$X = \cos\theta/M_R, \quad Y = \rho c\cos\theta'/(\rho' c' M_R), \qquad (1.9)$$

where M_R is the real part of M, Eq. (1.7).

It is evident from Eq. (1.8) that the absorption coefficient D differs markedly from zero when $X \sim 1$ while $Y \lesssim 1$. In particular, the maximum absorption coefficient occurs when

$X = \sqrt{2}$, $Y = 0$, or with consideration of Eq. (1.9),

$$\cos\theta/M_R = \sqrt{2}, \quad \rho c \cos\theta'/(\rho' c' M_R) = 0. \tag{1.10}$$

Then

$$D = D_{\max} = 2(\sqrt{2} - 1) \approx 0.83. \tag{1.11}$$

Thus, the largest value of the absorption coefficient which can exist for the passage of a sound wave through the phase boundary between two liquids is not related to any properties of the liquids or the frequency of oscillation, but is given by Eq. (1.11). This result was obtained for the special case of sound reflection from a rigid wall in [3]; it can be seen, however, that it remains valid for the general case of sound transmission through the phase boundary between arbitrary liquids.

The second equation of conditions (1.10), which when satisfied produces a maximum absorption coefficient, Eq. (1.11), can be satisfied in two ways: first, if $\rho/\rho' = 0$ (where we arrive at the case considered in [3]); second, if $\cos\theta' = 0$ (in this case we have $c'/c \geqslant 1$, and the angle of incidence is equal to the critical θ_{cr}). In accordance with this we rewrite the first condition of Eq. (1.10) in the form

$$\sqrt{1 - (c/c')^2} = M_R\sqrt{2}. \tag{1.12}$$

Thus, the maximum absorption coefficient Eq. (1.11) is attained at the critical angle of incidence, if Eq. (1.12) is satisfied. Since no limitations are placed on the quantity ρ/ρ' here, as follows from Eq. (1.7) the value of M_R may be large in comparison to unity (if $\rho/\rho' \gg 1$), and then Eq. (1.2) can be satisfied at critical angles of incidence far from $\pi/2$. This is a significant difference between the present case and the case where $\rho/\rho' = 0$ [where M_R is small, and thus, as is evident from the first condition of Eq. (1.10), the angle of incidence must be close to $\pi/2$].

We will note the unique symmetry of the phenomena of attaining maximum absorption coefficient (1.11) upon the onset of total internal reflection ($\cos\theta' = 0$) with respect to the same phenomenon at $\rho/\rho' = 0$.

If $\rho/\rho' \to \infty$, then it follows from Eqs. (1.7)-(1.9) that $D \to 0$. Thus in reflection of sound from a free surface absorption of sound energy does not occur.

2. At supercritical angles of sound incidence $c'/c \geqslant 1$ and $\theta \geqslant \theta_{cr}$. In order to obtain an expression for the reflection coefficient, in Eq. (1.5) we perform the replacement

$$\cos\theta' = i\sqrt{(c'/c)^2 \sin^2\theta - 1} = in,$$

where n is a real positive quantity. Since in the case of complete internal reflection radiation of sound energy into the lower medium does not occur, for the absorption coefficient we have $D = 1 - A_2\bar{A}_2$, or

$$D = 4X/[(1 + X)^2 + (1 + Y)^2], \tag{2.1}$$

where $X = \cos\theta/M_R$; $Y = \rho c n/(\rho' c' M_R)$. As before, the highest value of absorption coefficient will be $2(\sqrt{2} - 1)$ and is reached at $X = \sqrt{2}$, $Y = 0$. Combining this result (for supercritical angle of incidence) with that obtained above (subcritical angle of incidence), it can be said that when Eq. (1.12) is satisfied the absorption coefficient, considered as a function of angle of incidence, reaches its maximum value of $2(\sqrt{2} - 1)$ when the angle of incidence passes through the critical value.

The author thanks T. P. Zhizhina for much assistance in the study.

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